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To cite this article: Supapap Patsinghasanee, Ichiro Kimura, Yasuyuki Shimizu & Mohamed Nabi (2018) Experiments and modelling of cantilever failures for cohesive riverbanks, Journal of Hydraulic Research, 56:1, 76-95, DOI: [10.1080/00221686.2017.1300194](https://doi.org/10.1080/00221686.2017.1300194)

To link to this article: <https://doi.org/10.1080/00221686.2017.1300194>



Published online: 17 May 2017.



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Research paper

Experiments and modelling of cantilever failures for cohesive riverbanks

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ABSTRACT

This paper presents experiments and modelling of cantilever failures for a cohesive riverbank. Two types of cohesive materials with different bank cohesions were investigated under similar hydraulic conditions to clarify and simulate the failure phenomena. The experimental results showed that fluvial erosion undermined the banks during the initial stage of failure, and tension cracks then developed at the upper surface of the banks. Beam failure occurred thereafter. Additionally, a numerical model was developed using a triple-grid approach to simulate the behaviour of a cantilever within the framework of fluvial erosion, cantilever failure, slump block, and sediment transport. Moreover, cantilever failure was analysed using a limit equilibrium method to compute the stability of the overhanging block. The numerical model exhibited good agreement with the experimental results in terms of the spatial-averaged bank width along the banks and of the dominant failure type, which was beam failure.

Keywords: Channel forms; cohesive sediments; erosion processes; hydraulic models; laboratory studies; morphodynamics

1 Introduction

Riverbank failure is a key process in river morphodynamics, affecting a wide range of physical, ecological, and socio-economic issues related to alluvial channels; for example, loss of agricultural land, turbidity problems, and contamination problems (Rinaldi & Nardi, 2013; Rinaldi, Mengoni, Luppi, Darby, & Mosselman, 2008). However, because cantilever failures involve rapid channel widening and deliver large volumes of sediment into the alluvial channel, such failures are a serious issue in river morphology (Dapporto, Rinaldi, Casagli, & Vannocci, 2003; Nardi, Rinaldi, & Solari, 2012). Therefore, it is important to elucidate and model the mechanism underlying cantilever failure in order to gain a complete understanding of fluvial erosion and cantilever failure along an alluvial channel. This understanding enables problems of sediment transport, river ecology, renaturalization and the like to be addressed.

In riverbank stability analyses, previous researchers have been able to analyse a continuous bank failure owing to instability caused by the angle of repose (ASCE, 1998; Iwasaki, Shimizu, & Kimura, 2012; Jang & Shimizu, 2005; Nagata, Hosoda, & Muramoto, 2000; Thorne, 1982). Their studies involved the following processes: (1) bed scouring at the side bank, (2) bank collapse due to instability of the scoured bank, (3) deposition of the collapsed bank materials at the bank-toe, and (4) transport of the deposited materials. In contrast, cantilever failure has been studied based on a limited equilibrium method in which stabilizing and destabilizing forces are defined (Abam, 1997; Micheli & Kirchner, 2002; Pizzuto, 1984a, 1984b; Thorne & Tovey, 1981; Van Eerd, 1985). A limited number of cantilever failure studies have applied stability analyses based on the safety factor of the portion with cantilever failure. From this, three types of possible cantilever failure mechanisms – shear failure, beam failure, and tensile

Received 24 March 2016; accepted 21 February 2017/Open for discussion until 31 August 2018.

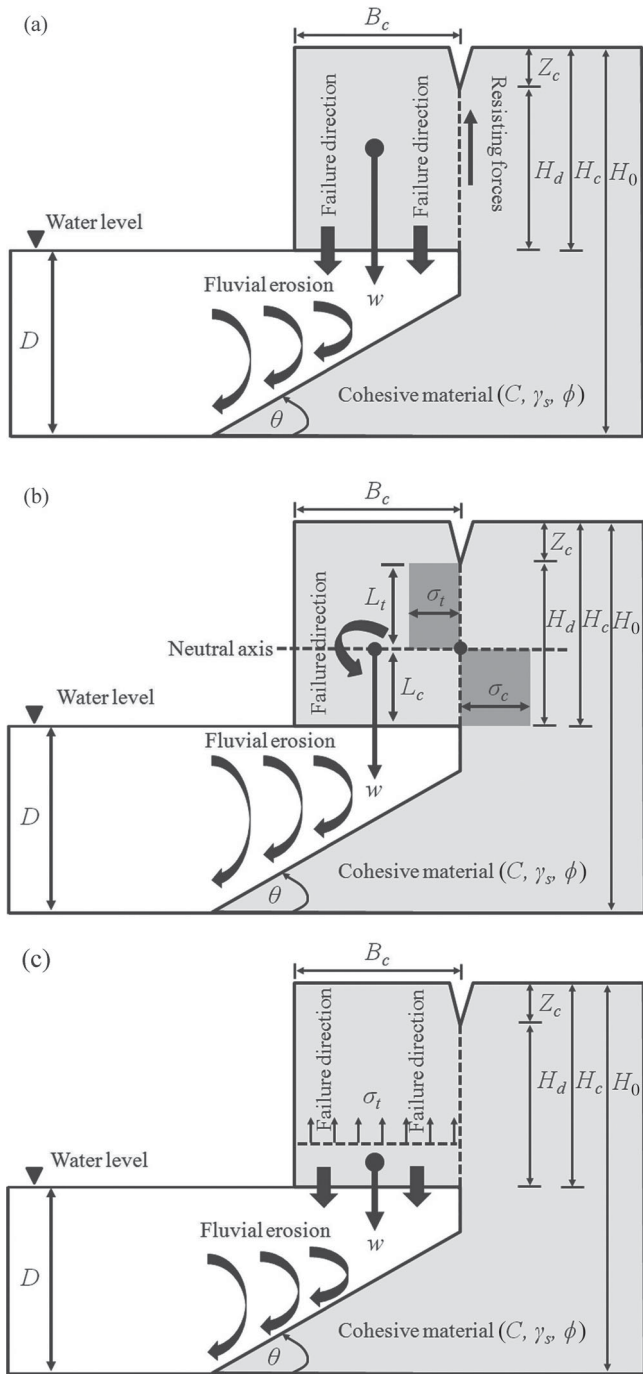


Figure 1 The three types of possible cantilever failures: (a) shear failure (b) beam failure and (c) tensile failure

failure – have been defined (Abam, 1997; Thorne & Tovey, 1981). Shear failure is expected to occur when the shear stress along the failure plane from the weight of the overhanging block (w) exceeds the resisting force (cohesion of the bank, C), as shown in Fig. 1a. Additionally, beam failure is related to unstable overhanging blocks rotating from the bank laterally into the channel. This occurs when rotational moment at the neutral axis is caused by the weight of the overhanging block; therefore, failure exceeds the restoring moment of bank cohesion (Fig. 1b). Moreover, tensile failure across a horizontal plane at a certain

height above the base causes the lower part of the overhanging block to recede (Fig. 1c). This occurs when the tensile stress due to the weight of the lower part of the overhanging block exceeds the tensile strength of the bank.

Moreover, the potential effects of hydrostatic pressure under the water surface of the main flow, the pore water pressure due to the groundwater level, and the hydrostatic pressure resulting from water present in the tension cracks were accounted for in the stability analysis of an overhanging block in shear failure and beam failure (Samadi, Amiri-Tokaldany, Dovoudi, & Darby, 2011). In addition, the functions of the uniform compressive strength and the tensile strengths of the cohesive materials were considered for modelling the beam failure based on Thorne and Tovey's formula (Thorne & Tovey, 1981) derived by Micheli and Kirchner (2002). In contrast, Van Eerd (1985) derived the non-uniform compressive and tensile strengths of a salt marsh cliff in order to model beam failure.

Several previous cantilever failure experiments have been reported without scale or sidewall correction effects, including experiments examining fluvial erosion rates and cantilever mechanisms (Fukuoka et al., 1999) and those investigating flow velocity and fluvial erosion rate (Fukuoka, Watanabe, Yamagata, & Kashiwagi, 2000). Moreover, experimental studies on the failure mechanism of cantilevers have also been reported in scale experiments that considered the scale and the sidewall correction effects. For instance, an experimental study on curved natural channels with an artificial overhanging block was conducted to determine flow characteristics, fluvial erosion rate, and bed deformation (Fukuoka, Ishikawa, Hibino, & Shimamoto, 1996). Additionally, Nardi et al. (2012) carried out experiments to investigate mass wasting failure in a sandy gravel bank and showed the occurrence of various failure processes, such as cantilever, slab, and slide failures. Samadi, Amiri-Tokaldany, Dovoudi, and Darby (2013) conducted experimental studies to determine the dominant cantilever failure mechanisms, which were found to be beam failure and tensile failure. However, these experimental studies (Samadi et al., 2013) formed the artificial overhanging block by manually removing bank materials from the bank-toe prior to the experiments. Moreover, Francalanci, Bondoni, Rinaldi, and Solari (2013) carried out experimental studies to observe the bank retreat processes during the tidal cycle and found various processes, including particle erosion, cantilever, and slide failures. Recently, investigations on the interaction between fluvial erosion and cantilever failure were conducted in an experimental flume, without consideration of the scale or sidewall correction effects (Patsinghasanee, Kimura, Shimizu, & Nabi, 2017).

For cantilever failure modelling, several numerical models have been developed to elucidate the complex mechanisms of cantilever failure. For example, bank stability models of shear failure and beam failure evaluated the sensitivity of parameters affecting cantilever failure. The results showed that, in analysing cantilever stability, the overhanging block dimensions and the cohesion of the bank are highly significant (Samadi et al., 2011).

A stress-strain behaviour model of cantilever failure was also applied to simulate the subsequent failure of an overhanging block by employing the limited equilibrium method (Samadi et al., 2013). A theoretical model of beam failure in salt marshes that incorporates the effects of the dynamic behaviour of an overhanging block was proposed. Such a block is identified by the presence of a tension crack and is subjected to hydrodynamic forces generated by wind waves, resistive forces resulting from the block overhang, and the cohesion of the bank (Bendon, Francalanci, Cappiotti, & Solari, 2014). Additionally, the numerical results of the cantilever and the continuous bank failure under two different conditions were compared to each other (Patsinghasanee, Kimura, & Shimizu, 2015). The comparison showed that the continuous bank failure model could not replicate the cantilever failure mechanisms of the bank. Furthermore, a coupling model of fluvial erosion and cantilever failure was proposed to simulate the subsequent cantilever failure (Patsinghasanee et al., 2017). The main limitation of this previous study was in the quantification of the fluvial erosion rate (ε) derived from the erodibility coefficient (k_d), because the determination of k_d remains complex and is dependent on several factors, including water content, bank cohesion, and silt-clay content (Couper, 2003; Grabowski, Droppo, & Wharton, 2011; Julian & Torres, 2006). Thus, in determining ε , the relationship between k_d and critical shear stress (τ_{bc}) needs to be obtained locally.

Following cantilever failure, the detached blocks crumble to form shapes similar to those of slump blocks, and cover the bank-toe. It is thought that the fluvial erosion rate decreases because of the failure of slump blocks (Dulal, Kobayashi, Shimizu, & Parker, 2010). A simplified model of a slump block was developed in order to replicate the meandering evolution process in experiments (without considering scale and sidewall correction effects) (Dulal et al., 2010) and in natural rivers (Parker et al., 2011). These previous studies employed a continuous bank failure concept in which the slump block was considered after bank collapse. Bank collapse was assumed to occur if the angle of the bank exceeded the submerged angle of repose. In such models, when the bed channel near the bank erodes and the cross-sectional gradient of the bank slope becomes steeper than the angle of repose, a slump block is assumed to be generated. Following this, the slump block is deposited at the bank-toe and the bank is then armoured. However, the continuous bank failure concept cannot replicate the complex mechanisms of shear failure and beam failure defined by Abam (1997) (Patsinghasanee et al., 2015). Therefore, in simulating cantilever failure phenomena, the previous slump block concepts cannot be applied directly (Dulal et al., 2010; Parker et al., 2011).

To address the limitations of the previous experiments and to model the cantilever failure process, a series of scale experiments were conducted with the objective of fully elucidating the complex mechanisms of cantilever failure by considering geometrical and material scaling, and the sidewall correction

effect. Additionally, the slump block failures that occur during the progress of cantilever failure, and the phenomena associated with the decomposition of those blocks, are discussed with reference to an experimental flume. Furthermore, a novel coupled numerical model that incorporates the effects of fluvial erosion, cantilever failure, slump block occurrence, and sediment transport was developed to simulate the cantilever failure mechanism.

2 Methodology

This section provides an overview of the scale experimental setup and a description of the novel numerical model of cantilever failure.

2.1 Experimental set-up

There are certain deficiencies in studies on the mechanisms underlying cantilever failure with a slump block, and there is a general lack of experimental literature. To address this, scale experiments on fluvial erosion and cantilever failure were conducted in a straight, rectangular flume with a fixed bed and a Plexiglas wall for real-time observation, and the temporal changes in the bank and the bed shapes were recorded. The water and the failure materials were recirculated using a constant head tank of water placed at the upstream end of the flume. Moreover, to replicate a steady uniform flow, the water discharge was kept constant at 6.45 l s^{-1} by using a rectangular weir to regulate the discharge during the experiments. In addition, the initial water level was set to zero, and a free-flowing condition was established at the downstream end. Furthermore, the initial bed of the flume was not covered by sediment, and sediment was not added from the sediment feeder.

Generally, the traditional limitations of the previous experimental studies were those of scale and sidewall correction effects. In the present study, the geometrical and material scaling, and the sidewall correction effect were considered in the establishment of the banks and the construction of the experimental flume. Therefore, the following equation was used as the equation governing the shear failure of an overhanging block (Muir Wood, 2004; Samadi et al., 2013):

$$f_{ss} = f(C/(\gamma_s B_c), H_c/B_c) \quad (1)$$

where f_{ss} is the safety factor of shear failure, C is the bank cohesion, γ_s is the unit weight of soil, B_c is the overhanging block width, and H_c is the overhanging block height. Here, the tensile strength and the inner friction angle are neglected because we assume a vertical failure section, and only the force acting tangentially to the vertical failure section is considered for shear failure.

Based on Eq. (1), if an overhanging block forms in the laboratory flume, the values of the two dimensionless parameters

must be similar between the prototype and the experimental models. Moreover, the overhanging block height affects both the resisting and driving forces acting on the unit width of an overhanging failure block (Muir Wood, 2004; Samadi et al., 2013). Therefore, this parameter can be neglected in the stability analysis, as shown in Eq. (2). Thus, it is necessary to evaluate only the remaining three parameters (i.e., C , γ_s and B_c) to model cantilever failure in the laboratory, i.e.:

$$f_{ss} = C/(\gamma_s B_c) \quad (2)$$

Based on the above explanation, f_{ss} was assumed to be equal to one. The values of γ_s and C for the materials were determined using a water pycnometer and a direct shear test. Before the experiment, the materials were tested to determine the cohesion of the bank and the internal friction angle by using a direct shear device, based on the ASTM D3080-98 standard test method for direct shear test of soil under consolidated drained conditions (ASTM, 1998). The direct shear test was conducted by gradually

increasing the normal stress by means of ASTM D2435-96 (ASTM, 1996). The soil samples were carefully trimmed to fit within the shear box dimensions and were placed in the shear box. Moreover, the materials were consolidated through normal stress under loads of 5, 10, and 20 kPa for 24 h. The horizontal and vertical deformations and the corresponding applied shear stresses were then recorded simultaneously. Further information regarding direct shear testing is given by Patsinghasanee et al. (2017). Following this, the overhanging block width was estimated by substituting f_{ss} , γ_s , and C into Eq. (2). As a result, the critical overhanging block width was found to be equal to 0.23 m. For the experimental design, the flume had a width of 0.8 m, a length of 8.0 m, and a height of 0.4 m, and the channel slope was set to 0.001, as shown in Fig. 2. In the upstream region of the flume (0.0–3.0 m), a wooden board was installed to limit fluvial erosion (between sections I and II in Fig. 2). At the middle reach of the flume (3.0–5.0 m), a bank was set (between sections II and III in Fig. 2). The bank was 0.5 m wide, which is approximately twice as large as the estimated value (0.23 m),

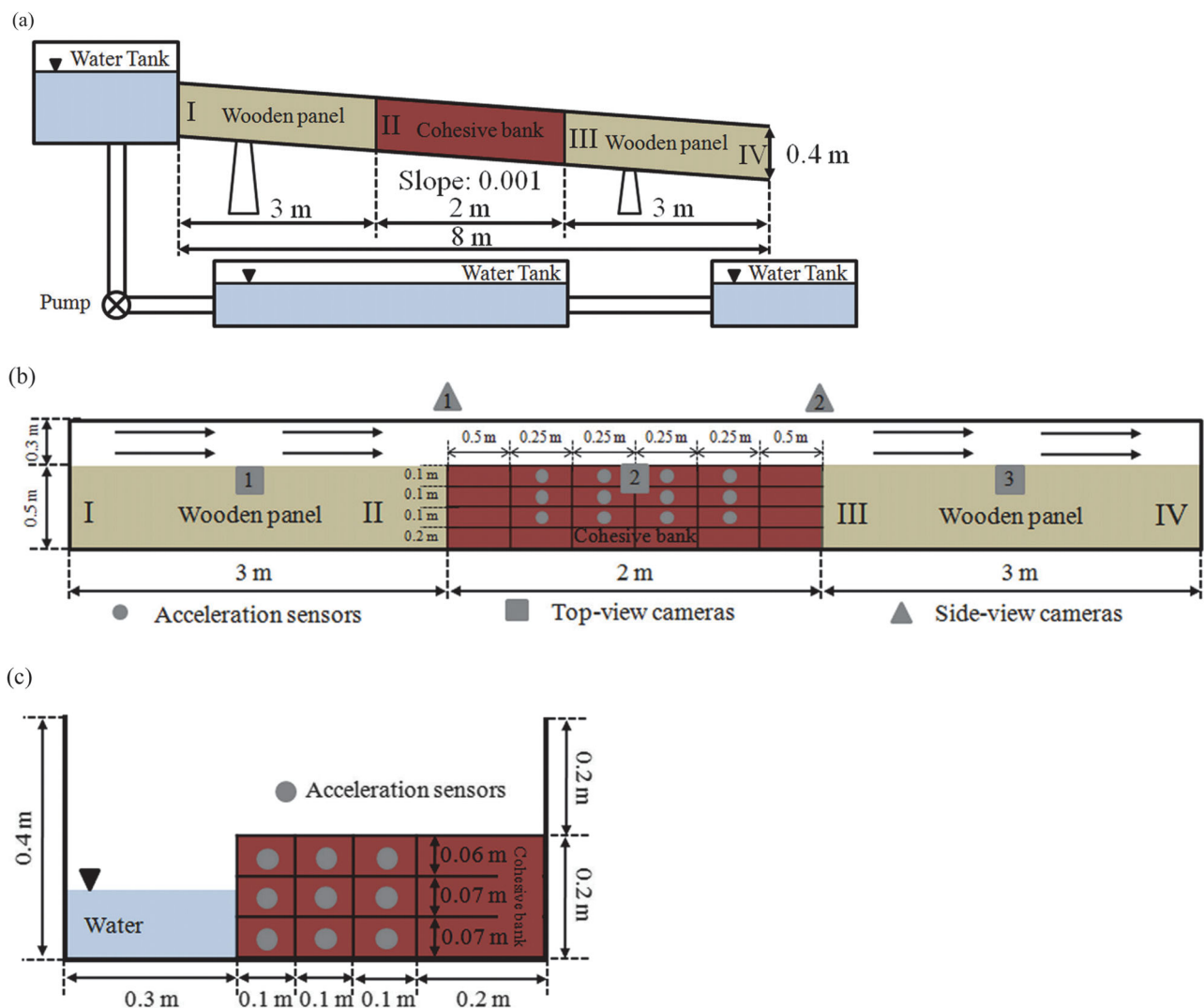


Figure 2 Dimensions of the experimental channels. (a) Overview of the experimental set-up; (b) top view of the experimental channel; (c) cross-section of the experimental channel

with a length of 2.0 m and height of 0.2 m. Downstream of the bank, a second wooden board was installed to protect the bank from fluvial erosion (between sections III and IV in Fig. 2).

For the sediment preparations, sediment composed of sand and silt with mean diameters (D_{50}) of approximately 0.23 mm and 28.4 μm , respectively, was used. The sediment was well mixed with water and was massaged manually. The sediment mixtures were initially wetted with water to achieve a water content (p_{wc}) of 32.2% and 39.6% for percentages of silt–clay content (p_{sc}) of 20% and 30%, respectively, under conditions that were similar to those of the previous study (Patsinghasanee et al., 2017). Two parameters (p_{sc} and p_{wc}) were varied in this study, as the cohesion of the bank is related mainly to the silt–clay content (Couper, 2003; Dulal & Shimizu, 2010; Grabowski et al., 2011; Julian & Torres, 2006) and the water content (Couper, 2003; Grabowski et al., 2011; Rinaldi & Nardi, 2013; Thorne & Tovey, 1981). However, the sediment mixtures were prepared with varying silt–clay contents, because for the sediments used in this study, it was difficult to control the water content for the different silt–clay contents. This is in accordance with the suggestion of a previous study (Patsinghasanee et al., 2017).

To allow for sufficient consolidation, the banks were compacted by applying a static load of 0.04 kg cm^{-2} for 72 h, a value similar to the static load acting on a bank (Nardi et al., 2012; Patsinghasanee et al., 2017). Moreover, the thickness of the compacted layer was 20 cm in each case. During the construction of the bank, a wooden panel was positioned to form a vertical bank. The panel was removed prior to the experiments. Dynamic compaction was not used for the protection of the Plexiglas wall from damage nor for the replication of the natural bank conditions that normally occur through static compaction. For material conditions, the values for bank cohesion and internal friction angle were the same as those of a previous study (Patsinghasanee et al., 2017) were used because the cohesive materials were mixed under conditions similar to those of the previous work. The bank geometries, discharges, and cohesive material properties used in the present experiments are summarized in Table 1.

The advantage of the experiment was the possibility of observing and recording the cantilever failure mechanism from the top and side views, and from inside the banks of the experimental flume during fluvial erosion, tension cracking, cantilever failure, and slump block formation. All failure mechanisms were recorded using five high-resolution video cameras, and 36 acceleration sensors, as shown in Fig. 2b and c. The video cameras were fixed perpendicularly in position for the top view (three cameras) and side view (two cameras) of the flume to record the temporal changes in bank width and water depth, respectively. Additionally, metric scales were positioned in the horizontal and vertical directions of the experimental flume. The video frames were subsequently converted into greyscale still images, and the temporal changes in bank width and water depth were determined through a digitization process, with a precision

Table 1 Summary of the experimental conditions

Variable	Symbol	30% of p_{sc}	20% of p_{sc}
Discharge (ls^{-1})	Q	6.45	6.45
Initial bank height (m)	H_0	0.20	0.20
Initial bank width (m)	B_0	0.50	0.50
Initial water depth (m)	D_0	0	0
Channel slope (–)	i	0.001	0.001
Silt–clay content (%)	p_{sc}	30	20
Water content (%)	p_{wc}	39.6	32.2
Bank cohesion (Pa)	C	5.76	6.41
Internal friction angle ($^\circ$)	ϕ	34.3	41.3
Bank critical shear stress (Pa)	τ_{bc}	0.57	0.38
Erodibility coefficient ($\text{m}^3 \text{N}^{-1} \text{s}^{-1}$)	k_d	105.6	42.8
Reynolds number (–)	R	21,500	21,500
Froude number (–)	F	0.56	0.56

of approximately 1 mm. For the scale experiments, an important task was to clarify the cantilever failure processes step by step. Therefore, acceleration sensors were installed inside the banks. These sensors recorded the catastrophic failure mechanisms and identified the fluvial erosion or cantilever failure at points where the values change greatly. The acceleration sensors were installed inside the banks uniformly in a way similar to that of previous laboratory experiments (Shimada, Watanabe, Yokoyama, & Tsuji, 2009). The similar method of using the acceleration sensors was reported in a large-scale experiment of a riverine levee breach (Kakinuma & Shimizu, 2014). Acceleration signals in the x_{axis} , y_{axis} and z_{axis} directions were produced by the failure and outflow of the installation positions inside the banks. These sensors are self-recording, and they had to be collected after the experiment for data acquisition.

2.2 Numerical model

The main objective of the present coupling model is to replicate the physical mechanisms of fluvial erosion, tension crack, cantilever failure, slump block generation and decomposition, and sediment transport of the banks.

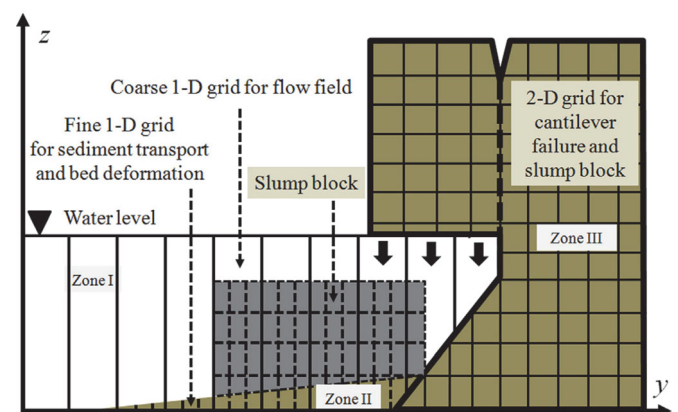


Figure 3 Framework of the triple-grid approach to the cantilever failure model considering the slump block

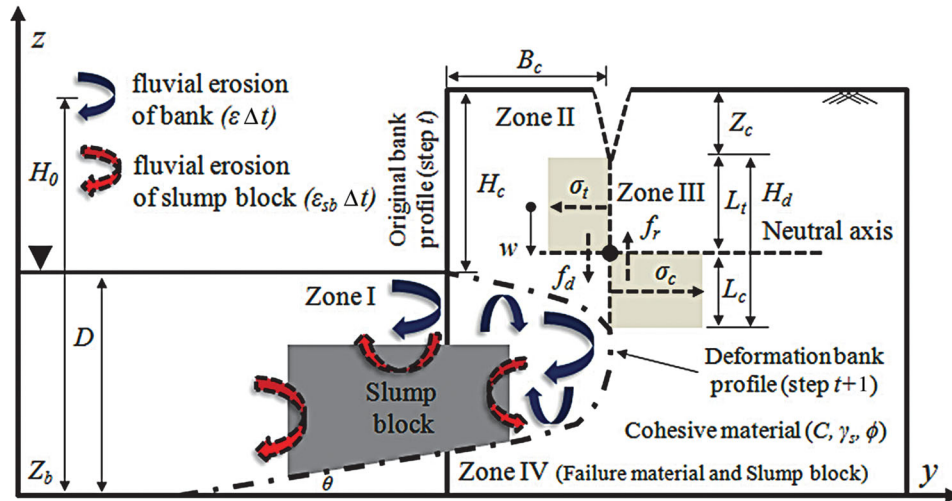


Figure 4 Overhanging geometry considering the slump block, and forces exerted on the incipient failure block

A cantilever failure model was developed by using a triple-grid approach. This approach involved the use of a coarse one-dimensional (1-D) grid, which is a large calculation grid cell (see Zone I in Fig. 3), for the flow field in the lateral direction; a fine 1-D grid, which is smaller calculation grid cell (see Zone II in Fig. 3), for sediment transport and bed deformation in the lateral direction; and a 2-D grid for cantilever failure, and slump block generation and decomposition in the vertical and lateral directions (see Zone III in Fig. 3). During the initial stage of the computation (Fig. 4), the model reproduced fluvial erosion at the lower part around the wetted perimeter of the bank, which is shown by the dot-dash line in Zone I. Next, tension crack development in Zone II and cantilever failure in Zone III (the dashed vertical line along the bank) were captured. Cantilever failures were defined by considering the factor of safety for the two types of failure (i.e. based on shear failure and beam failure mechanisms); tensile failure was omitted because such failure has rarely been observed on natural rivers (Darby, Rinaldi, & Dapporto, 2007). For beam failure, the failure block rotated 90° anti-clockwise into the channel and dropped at the bank-toe (Fig. 5a). Therefore, the pivot point for rotation of the failure block after detachment from the bank was at the lowest edge of the failure plane. In contrast, the overhanging failure block came to rest directly at the intermediate drop point in shear failure (Fig. 5b). The slump block decomposition process then developed at the bank-toe in Zone IV (Fig. 4). This approach involves applying four sub-models – for fluvial erosion, cantilever failure, slump block generation and decomposition, and sediment transport – at each set of discrete time steps. A logic diagram of the computational sequence used for the coupled mechanism is illustrated in Fig. 6.

2.3 Fluvial erosion rate

The flow field was calculated using a uniform flow model on a coarse lateral 1-D grid cell (Process 1 in Fig. 6). To avoid

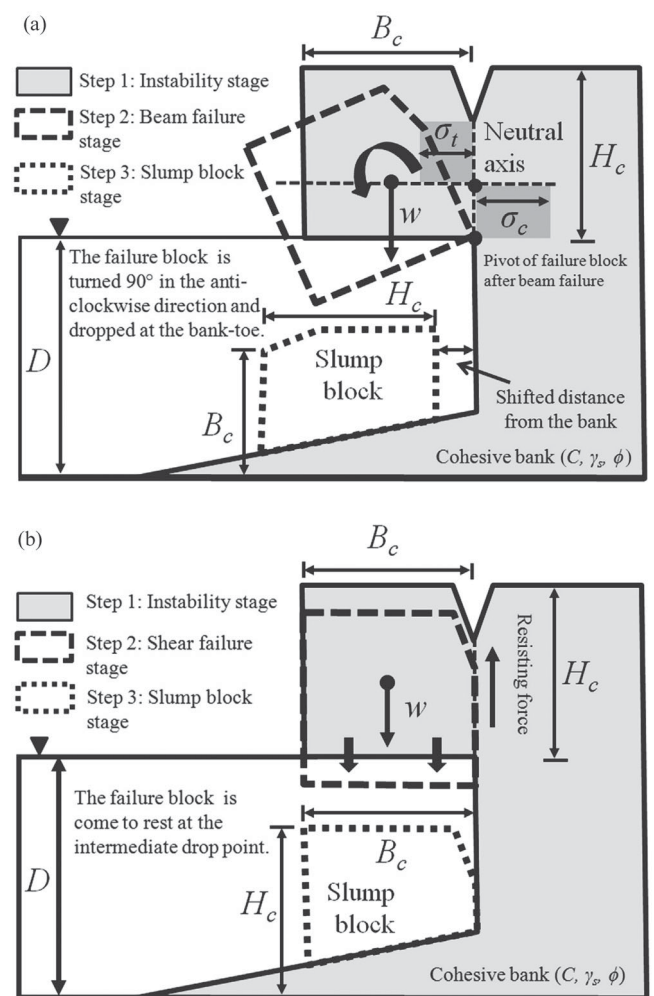


Figure 5 Slump block locations and dimensions of two different cantilever failure types: (a) beam failure; (b) shear failure

the small grid-cell width in the vicinity of the bank, the grid cell division was defined from the edge of the bank instead of from the left side of the computational domain. Therefore, the flow field equation can consider the complicated shape of fluvial

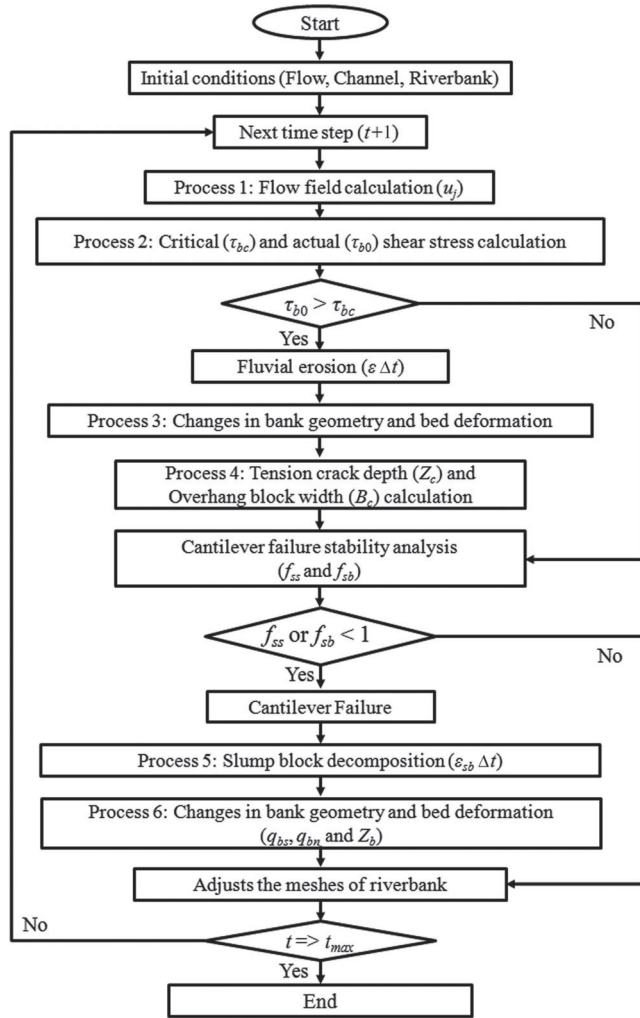


Figure 6 Logic diagram of the computational sequence

erosion near the bank by evaluating the hydraulic radius in a coarse lateral grid cell. The velocity in each calculation cell (u_j) is computed as:

$$u_j = \frac{1}{n} R_j^{2/3} i^{1/2} \quad (3)$$

where n is the Manning roughness parameter along the channel ($0.011 \text{ m}^{-1/3} \text{ s}$) calculated using the Manning–Strickler equation ($k_s^{1/6}/7.66g^{1/2}$), g is gravity acceleration (9.81 m s^{-2}), k_s is the roughness height defined as $(1 \text{ to } 3)D_{50}$, R_j is the hydraulic radius in each calculation cell (A_j/P_j), A_j is the cross-sectional area of a cell, P_j is the perimeter of a cell, i is the bed slope, and j is the number of the lateral cell. The effect of the slump block on the flow is considered only as the change in the hydraulic radius. The present model cannot consider the drag force imparted by the slump block because the model assumes a uniform channel shape and flow in the streamwise direction. Currently, this approximation is reasonable because the observed slump blocks extend in the streamwise direction. However, the drag force imparted by the slump block seems to be more important in the case of real rivers because the assumption of a uniform

channel shape is not applicable. This matter should be carefully considered in future work focusing on phenomena in real rivers.

The fluvial bank erosion rate (Process 2 in Fig. 6) is quantified using an excess shear stress formula applied for a wetted perimeter of the bank, as given below (Arulanandan, Gillogley, & Tully, 1980):

$$\varepsilon = k_d (\tau_{b0} - \tau_{bc})^a \quad (4)$$

where ε is the fluvial erosion rate of the bank, k_d is the erodibility coefficient, τ_{b0} is the actual shear stress applied by the flow, τ_{bc} is the critical shear stress, and a is an exponent generally assumed to be one. In Eq. (4), the positive value of excess shear stress is only considered for the fluvial erosion estimation. Although Eq. (4) appears simple, the k_d for the bank is difficult to determine in the numerical model. Many factors, such as water content, bank cohesion, and silt–clay content, influence the bank erosion, making k_d difficult to quantify. The value of k_d was then determined via model calibration, by forcing the best agreement of the streamwise average bank widths between the numerical and the experimental results.

The actual bed shear stress was determined by the function of the hydraulic radius and the channel slope in each calculation cell, as shown in Eq. (5):

$$\tau_{b0} = \rho g R_j i \quad (5)$$

where ρ is the density of water (1000 kg m^{-3}). The empirical equation of the critical shear stress as a function of the percentage of the silt–clay content (p_{sc}) was employed as expressed in Eq. (6) (Julian & Torres, 2006):

$$\tau_{bc} = 0.1 + 0.1779 p_{sc}^3 + 0.0028 p_{sc}^3 - 2.34 \cdot 10^{-5} p_{sc}^3 \quad (6)$$

To characterize the fluvial erosion rate around the wetted perimeter of the bank, the fluvial erosion rate (Eq. (4)) was evaluated in each time step by the function of k_d , τ_{b0} and τ_{bc} to erode the bank. For the actual shear stress, this procedure is a simple method for evaluating the near-bank shear stress by neglecting the lateral shear stress. This is because the secondary current in the straight experimental flume presents velocities of approximately 2% of the streamwise velocity (Blanckaert, Duarte, & Schleiss, 2010; Nezu, Tominaga, & Nakagawa, 1993). Moreover, the effect of turbulent diffusion is negligible because mechanisms such as sliding and slump falling are dominant in experimental studies (Blanckaert et al., 2010). In addition, the shear stress distribution at the bank, instead of a constant shear stress, does not significantly change the simulation results (Motta, Langendoen, Abad, & Garcia, 2014). As a result, this procedure is an ideal estimation of the fluvial erosion rate; it was employed to simulate the coupled numerical model of the cantilever failure phenomena in a simple fashion.

For the computational process of fluvial erosion, the bank profiles were deformed in accordance with the fluvial erosion

simulated at the end of each discrete time step (Process 3 in Fig. 6). However, the shape and size of the computational grid cells were fixed. To express the continuous change of the bank shape with a fixed rectangular grid, we introduced a new variable: the undisturbed cell occupancy rate (Ω_t). It is defined in Eq. (7):

$$\Omega_t = \frac{(V_{org} - \sum_{t=1}^n V_{erd})}{V_{org}} \quad (7)$$

where V_{org} is the initial grid cell volume of a bank, and V_{erd} is the simulated fluvial erosion volume in each time step. This parameter takes a value between 0 (no cohesive material), and 1 (all cohesive material).

The shift of the bank boundary was modelled as follows. If the accumulated fluvial erosion volume was less than the initial grid cell volume of the bank, the boundary nodes were not shifted horizontally inward through fluvial erosion (Fig. 7a). Moreover, the boundary conditions did not change in this procedure. If the accumulated fluvial erosion volume was greater than or equal to the initial grid cell volume of the bank, then the new grid cells were assigned to update the new bank profile nodes (Fig. 7b).

2.4 Cantilever failure

The two cantilever failure mechanisms applied in this study have been described by Thorne and Tovey (1981) for shear failure, and by Micheli and Kirchner (2002) for beam failure (Process 4 in Fig. 6). In addition, the advantage of the present numerical model is that it couples fluvial erosion and cantilever failure with the two types of failure mechanisms, whereas previous numerical models only considered shear failure for replication of overhanging block failure (Darby et al., 2007; Motta et al., 2014; Rinaldi et al., 2008).

Shear failure involves shear stress along the vertical plane. Such failure is expected when the shear stress along the vertical plane of the overhanging block weight (f_d) exceeds the resistive force of the cohesive bank (f_r), as shown in Figs 4 and 5. The safety factor for shear failure (f_{ss}) is described as:

$$f_{ss} = \frac{CH_d}{\gamma_s A_b} \quad (8)$$

where H_d is the effective length and A_b is the cross-sectional area of the overhanging block. Moreover, the effects of pore pressures and hydrostatic confining pressures are neglected for this study, in accordance with the existing literature (Abam, 1997; Thorne & Tovey, 1981; Wang, Kuang, & Su, 2016).

Beam failure is related to unstable overhanging blocks, whose instability causes rotation around a neutral axis (Figs 4 and 5). Above this axis, failure is caused by tension; below the axis, failure is caused by compression. Failure occurs when the rotational moment of the overhanging block overcomes the resistive moments of the soil strength in the tension and the compression zone. The safety factor for beam failure (f_{sb}) can be

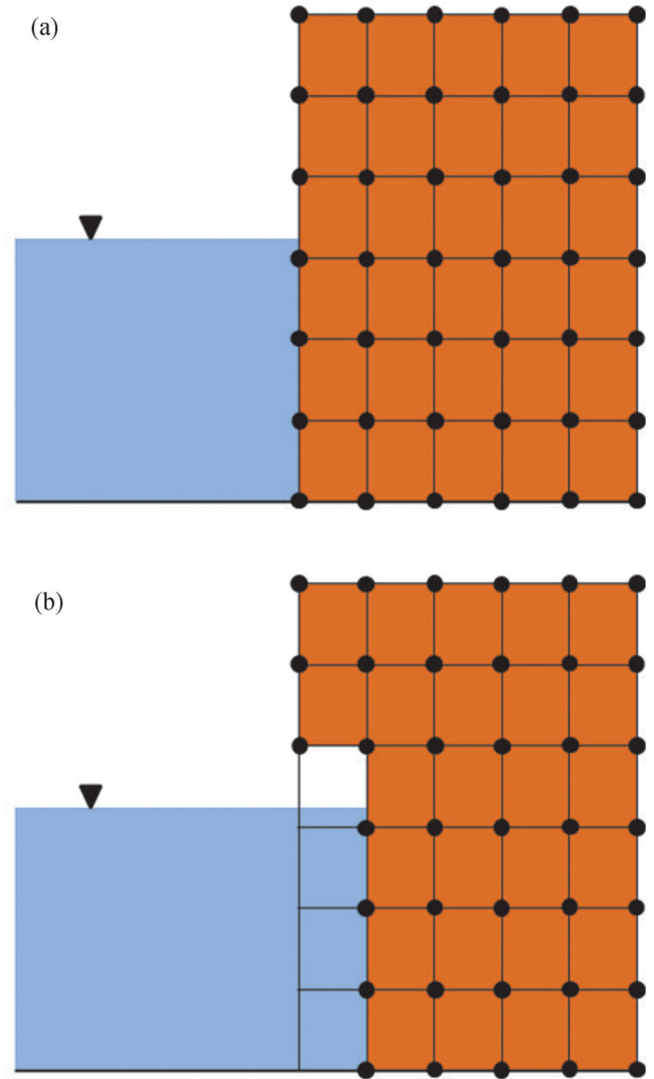


Figure 7 Illustration of the two procedures for the boundary nodes modelling the effect of fluvial erosion: (a) first procedure; (b) second procedure

described as:

$$f_{sb} = \frac{\sigma_c L_c^2 + \sigma_t L_t^2}{\gamma_s H_c B_c^2} \quad (9)$$

with $H_d = L_c + L_t$ and $\sigma_c/\sigma_t = L_t / L_c$, where L_c and L_t are the overhanging lengths under compressive and tensile strength, respectively, and σ_c and σ_t are the uniform compressive and tensile strengths, respectively. Previous studies employed the assumption of uniform profiles for compressive and tensile stresses acting on the failure plane (Micheli & Kirchner, 2002; Samadi et al., 2011; Thorne & Tovey, 1981). However, several studies applied non-uniform profiles for compressive and tensile stresses (Abam, 1997; Van Eerd, 1985; Wang et al., 2016). Additionally, Van Eerd (1985) compared the safety factor for beam failure to the non-uniform (Van Eerd, 1985) and uniform (Thorne & Tovey, 1981) profiles for compressive and tensile stresses. The comparison results showed that the safety factor

as calculated by Thorne and Tovey (1981) is approximately one and a half times that calculated by Van Eerd (1985).

To link the cohesion to the tensile strength of the cohesive material, the equation of Fang and Hirst (1973) and Fang and Daniels (2006) was employed as follows:

$$\sigma_t = \frac{C}{\xi} \quad (10)$$

where $\xi = 1/(0.34 + 0.01i_w)$, ξ is the ratio of tensile strength to cohesion, and i_w is the plasticity index. Finally, based on the Chen–Drucker modified Mohr–Coulomb criteria (Chen & Drucker, 1969), the uniform compressive strength was computed using:

$$\sigma_c = 2C \tan \left(45 + \frac{\phi}{2} \right) \quad (11)$$

where ϕ is the internal friction angle. The tension crack depth (Z_c) was assumed to be half of the overhanging block height because, typically, when ratios of tension crack depth to overhanging block height range from 0.3 to 0.7, the safety factor is not altered by more than 10% (Thorne & Abt, 1993).

2.5 Slump block

For the slump block, a novel scheme was proposed in this study. Following cantilever failure, the overhanging block immediately dropped into the channel in front of the bank and became a slump block. Here, the complexity of the slump block is implicitly represented by the following assumption. The slump block dimensions were assumed to be equal to the overhanging block dimensions (B_c and H_c). Additionally, the settled dimensions of the slump block differed between shear failure and beam failure (Fig. 5). Furthermore, the decomposition processes (Process 5 in Fig. 6) of the slump block were then induced by the governing fluvial erosion at the submerged surfaces of the slump block (Zone IV in Fig. 4). The erosion rate of the slump block was calculated as:

$$\varepsilon_{sb} = k_{sb} \varepsilon \quad (12)$$

where ε_{sb} is the fluvial erosion rate of the slump block and k_{sb} is the coefficient of the fluvial erosion rate (1.2). In this study, we assumed that the erosion strength of a slump block is smaller than that of the original bank, because the slump block experienced a disturbance owing to the drop. In addition, slump blocks may deflect high-velocity cores away from the bank, thus increasing shear stresses on the slump block surfaces, and may divert the flow up, over, and around their topography (Hackney et al., 2015; Motta et al., 2014; Parker et al., 2011). For these reasons, the coefficient of the fluvial erosion rate must be higher than that obtained by forcing the best agreement between the calculated and measured streamwise average bank widths.

2.6 Sediment transport and bed deformation

Following fluvial erosion, cantilever failure, and slump block decomposition, the failed materials were dropped or rolled down into the channel. The dropped materials decomposed into two different materials (sand and silt) (Process 6 in Fig. 6). The silt was assumed to become wash load, which moves with the water flow and no longer affects the bed form. The mass of sand of the failed material is equal to that of sand from the fluvial erosion and failed block. For this reason, the failure sand acts as bedload. In addition, the slope angle of the failure materials was equal to the angle of repose of the bank material (Dulal et al., 2010; Iwasaki et al., 2012; Jang & Shimizu, 2005; Nagata et al., 2000; Parker et al., 2011). The equations describing sediment transport and bed deformation are presented below.

The sediment transport rate in the streamwise direction (q_{bs}) was calculated using the formula proposed by Ashida and Michiue (1972):

$$q_{bs} = 17\tau_*^{3/2} \left(1 - \frac{\tau_c^*}{\tau_*} \right) \left(1 - \sqrt{\frac{\tau_c^*}{\tau_*}} \right) \sqrt{s_g g D_{50}^3} \quad (13)$$

where τ_* is the non-dimensional bed shear stress ($u_*^2/(s_g g D_{50})$), u_* is the shear velocity ($\sqrt{g R_j i}$), s_g is the specific weight of the sediment (2.65), and τ_c^* is the non-dimensional critical bed shear stress. The equation for non-dimensional critical bed shear stress proposed by Iwagaki (1956) is based on theoretical and experimental analyses of sediment particle incipient motion. The critical shear velocity derived from the equation is in good agreement with that from the Shields diagram. This equation is widely used in river engineering studies (Dulal et al., 2010; Iwasaki et al., 2012; Jang & Shimizu, 2005; Kakinuma & Shimizu, 2014; Patsinghasanee et al., 2015, 2017).

The sediment transport rate in the lateral direction (q_{bn}) is expressed as (Hasegawa, 1984):

$$q_{bn} = -q_{bs} \sqrt{\frac{\tau_c^*}{\mu_s \mu_k \tau_*}} \frac{\partial Z_b}{\partial y} \quad (14)$$

where μ_s and μ_k are the static (1.0) and kinetic (0.45) friction coefficients, respectively; Z_b is the bed elevation, and y is the coordinate component in the lateral direction. In this equation, the sediment transport rate in the lateral direction is assumed to be proportional to the lateral bed slope. The original form of Hasegawa's (1984) equation has an additional term related to the secondary current of the first kind. We can neglect this term because we are considering the case of a straight channel. Note that Hasegawa's (1984) equation is widely used for determining the sediment transport rate in the lateral direction (i.e. Dulal et al., 2010; Iwasaki et al., 2012; Jang & Shimizu, 2005; Kakinuma & Shimizu, 2014; Patsinghasanee et al., 2015, 2017). The bed deformation was calculated using a continuity equation of sediment transport in an orthogonal coordinate system and is

expressed as:

$$\frac{\partial Z_b}{\partial t} + \frac{1}{1-\lambda} \left(\frac{\partial q_{bn}}{\partial y} \right) = 0 \quad (15)$$

where t is time and λ is the porosity of the material (0.4).

3 Results

3.1 Experimental results

A summary of the cantilever failure and the slump-block processes during each experiment is provided, with reference to the images of the temporal changes in bank width, and with each component of the acceleration values to clarify the failure mechanisms inside the cohesive banks.

In our previous study, we conducted a series of experiments to test the cantilever failure process in terms of fluvial erosion, tension crack development, and cantilever failure for different silt-clay contents (Patsinghasanee et al., 2017). Our previous experiments were conducted in a straight rectangular flume. The flume was 30 cm wide, 10 m long, and 20 cm high, and the flume slope was set to 0.002. During the tests, the water and the sediment were recirculated using a head tank, and water was discharged into the bank with a constant discharge of 2.4–6.4 l s⁻¹. For instance, in these experiments we tested the possible sediment mixtures related to the percentage of silt-clay content (20% and 30%) that appeared to be appropriate for replication of the cantilever failure mechanisms. Moreover, this percentage of silt-clay content was consistent with that of a previous experimental study by Dulal and Shimizu (2010). A summary of the initial tests was reported in a previous paper (Patsinghasanee et al., 2017). The results of these experiments were used for comparison with, and analysis of, the failure mechanisms in scale experiments.

For scale experimental results, Fig. 8 shows the acceleration values in the x_{axis} , y_{axis} and z_{axis} directions recorded by the sensors. Here, the x_{axis} , y_{axis} and z_{axis} directions represent the streamwise, lateral, and vertical directions, respectively (Fig. 2). In terms of data interpretation, acceleration values that suddenly change in the y_{axis} direction or the z_{axis} direction indicate the occurrence of beam failure or shear failure, respectively, in a bank. Moreover, changed acceleration values in the x_{axis} direction indicate that the fluvial force erodes the bank at the submerged zone. (See the value in Fig. 8). The physical meanings of the positive and negative values are as follows. In the z_{axis} direction, accelerations with positive values are accelerations in the downward direction. In the y_{axis} direction, accelerations with negative values are accelerations towards the interior of the bed channel. In the x_{axis} direction, positive values denote the upstream direction. A summary of the failure processes determined in the experimental studies is provided below, with a reference photo given in Figs 9 and 10. The temporal changes in bank geometry along the bank measured using the acceleration

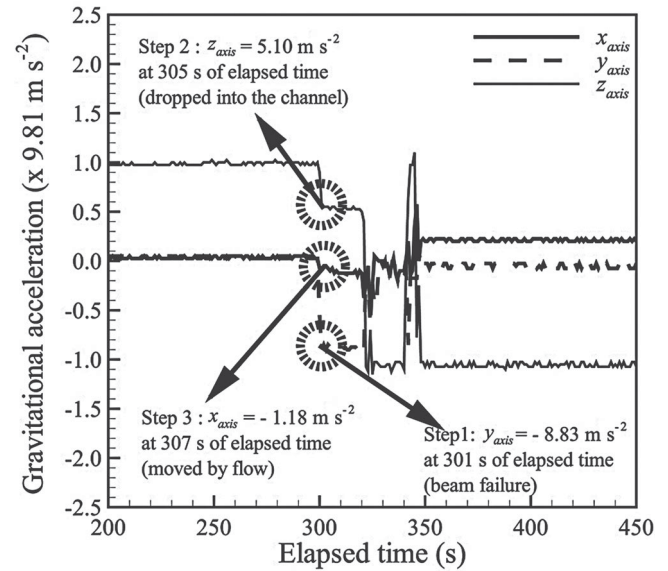


Figure 8 Examples of data interpretations of acceleration values in the case of 30% p_{sc}

sensors are shown in Fig. 11. Moreover, in the following experimental works, the tests were required to be stopped when an equilibrium stage was reached (no failure during a 3-h period).

In the case of 30% p_{sc} , fluvial erosion occurred immediately after the experiment started. The first considerable tension crack (Fig. 9a) and beam failure (Fig. 9b) were observed at a distance of 7.4 to 134.7 cm from the upstream end at 296 and 300 s. Moreover, the failure block had a width of 12.9 cm and a length of 127.3 cm. For the first failure, the decomposition time, which is the time required for complete decay of the slump block on the bed channel, was 163 s. Following the beam failure, the bank slope was reshaped toward an angle of 90°. A large tension crack and beam failure were then observed at 119.9 to 200.0 cm from the upstream end at 461 s. The slump block of this failure had a width of 14.3 cm, a length of 80.0 cm, and a decomposition time of 55 s. From 1560 to 9420 s, six successive tension cracks and beam failures randomly developed along the bank. In addition, the decomposition time for the six successive failure blocks was within the range of 1763 to 3981 s. The temporal changes in bank width for this case are shown in Fig. 10a. The experiment reached the equilibrium stage at 9420 s (Fig. 9c). The experiment is summarized in Table 2.

In the case of 20% p_{sc} , during the initial stage, fluvial erosion occurred along the bank; however, it became more evident in the downstream region from the start time to 1890 s. In the first and second cantilever failures, the experiment showed a backward erosion process from 144.1 to 191.8 cm (at 1890 s), and from 181.6 to 200.0 cm (at 3120 s) from the upstream end. The third and fourth beam failures then developed at the middle region of the bank from 33.3 to 138.3 cm from the upstream end at 3510 s and 3880 s. The fourth tension crack is illustrated in Fig. 9d, and the fourth beam failure is shown in 9e, and Fig. 11c by top-view video cameras and the acceleration sensors, respectively.

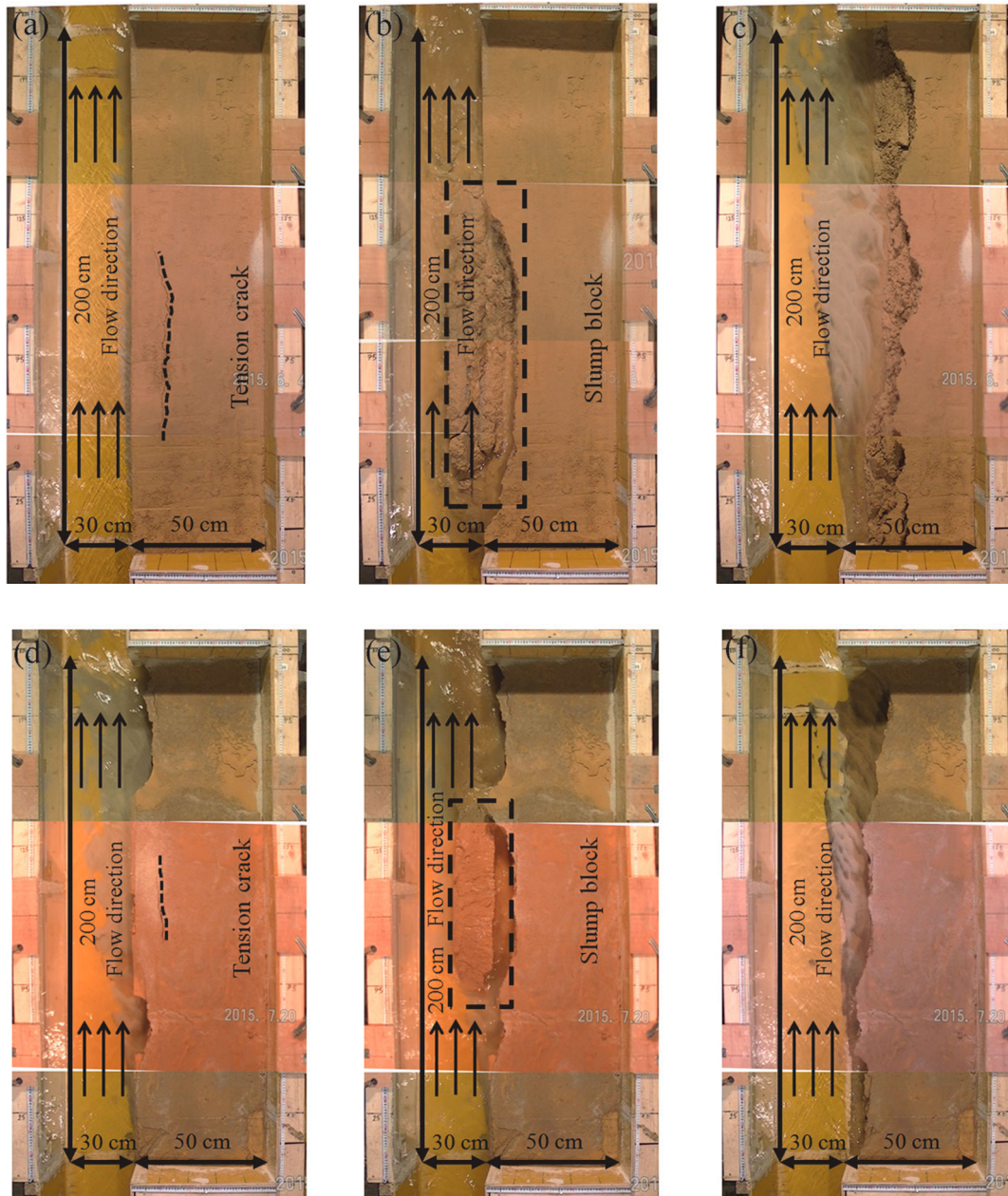


Figure 9 Example experimental results in the cases of 30% and 20% p_{sc} : (a) first tension crack (30% p_{sc}); (b) first beam failure and slump block (30% p_{sc}); (c) equilibrium stage (30% p_{sc}); (d) fourth tension crack (20% p_{sc}); (e) fourth beam failure and slump block (20% p_{sc}); (f) equilibrium stage (20% p_{sc})

The fifth ($t = 4770$ s) and sixth ($t = 5100$ s) beam failures were then observed from 122.8 to 168.8 cm from the upstream end, and from 157.9 to 200.0 cm from the upstream end, respectively. Additionally, the last two beam failures developed at the upstream end at 5505 s and 6600 s. Regarding the failure block dimensions, the width and length of failure blocks were within the range of 5.7 to 13.5 cm and 14.8 to 73.8 cm, respectively. The decomposition time for the eight beam failures varied from 61 to 904 s. The decomposition times in this case (20% p_{sc}) are smaller than those in the case of 30% p_{sc} because the failure block volumes in the case of 20% p_{sc} are slightly smaller than in the case of 30% p_{sc} . The temporal changes in bank width in the case of 20% p_{sc} are illustrated in Fig. 10b. The experiment

reached the equilibrium stage at 6600 s (Figs 9 and 11). Finally, the case of 20% p_{sc} is summarized in Table 3.

The experimental results show that fluvial erosion has the greatest effect on the occurrence of cantilever failure instability because such erosion increases the width of the overhanging block and reduces its stability. This process was captured using side view video cameras and the varied acceleration signal in the x_{axis} direction, as shown in Fig. 11b. This is because fluvial erosion is related to the loosening of weak bonds between the material particles. The observations from the present experiments are similar to those of previous studies (Abam, 1997; Darby et al., 2007; Patsinghasanee et al., 2017; Pizzuto, 1984a, 1984b; Rinaldi & Nardi, 2013; Thorne & Tovey, 1981). Tension

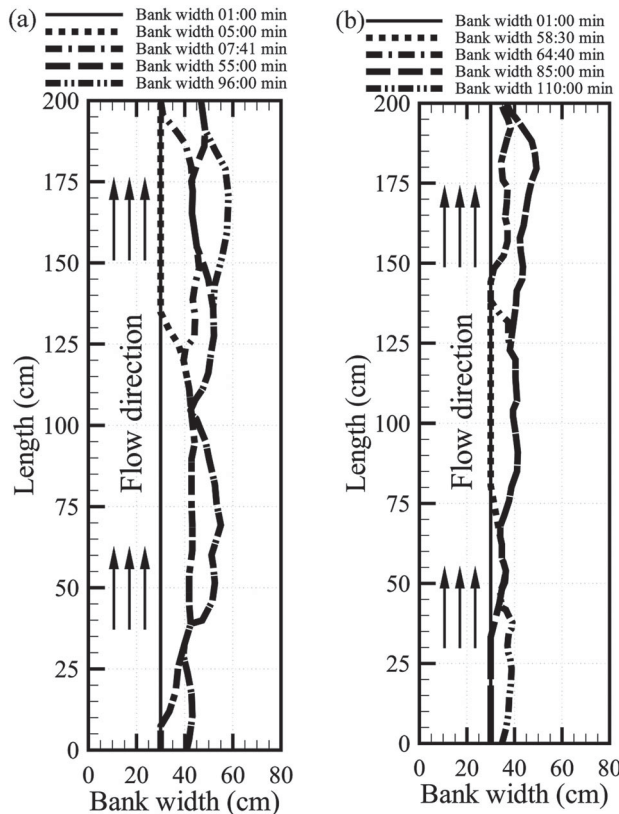


Figure 10 Experimental results of temporal changes in bank shape in the cases of 30% and 20% p_{sc} . (a) case of 30% p_{sc} ; (b) case of 20% p_{sc}

cracks then developed on the upper surface of the banks. Such cracks seemed to develop only when the cantilever was already close to failure. Following this, cantilever failure occurred along the tension crack line. For this reason, we assumed that the tension crack dimensions are equal to the overhanging block dimensions. The dominant failure mechanism was observed to be beam failure, which was identified using the acceleration signal in the y_{axis} direction, as shown in Fig. 11c. This is consistent with the findings of previous researchers, who have suggested that beam failure may be prevalent (Nardi et al., 2012; Patsinghasanee et al., 2017; Pizzuto, 1984a, 1984b; Samadi et al., 2013; Van Eerd, 1985). For the slump block, a single slump block or a series of slump blocks caused deposition at the bank-toe, which acts like a new bank. Then, the fluvial force eroded the slump block. This erosion caused higher shear stress on the bank and led to downstream enlargement of the eroded part. After this, the continued decomposition of slump blocks induced the creation of more slump blocks. Following this, the bank continued to erode until the erosion generated a dead zone, where the flow velocity was lower and fluvial erosion was inactive. Finally, the development zone of the new slump blocks might propagate to the upstream region. These observation results are in good agreement with the previous experimental results on the development of banks with slump blocks (Hackney et al., 2015).

3.2 Numerical results

Numerical simulations with durations of approximately 6 h (real time) were performed under the same conditions as in the experimental studies. In Fig. 12, the numerical and experimental results of the streamwise-averaged bank width are compared to a perfect-agreement line, and each symbol falls within the range of the 15% error line for the case of 30% p_{sc} and within the range of the 10% error line for the case of 20% p_{sc} . The streamwise-averaged bank width in the case of 30% p_{sc} was approximately 1.2 times that of the experimental results at nearly the equilibrium stage, as a result of the high bank protection owing to the armouring effect in the experimental flume. Moreover, the main error of the simulated streamwise-averaged bank width in the case of 20% p_{sc} occurred after the fourth failure, because the numerical model is limited in its ability to model the decomposition of large slump blocks.

The fluvial erosion rate for the case of 30% p_{sc} was higher than that for the case of 20% p_{sc} at 270 s under similar hydraulic conditions, as shown in Fig. 13; this indicates that banks with a higher p_{sc} (30%) are more susceptible to failure than those with a lower p_{sc} (20%). The observations of the present work are similar to those of previous studies in terms of changes in the silt-clay content (Couper, 2003; Dulal & Shimizu, 2010; Julian & Torres, 2006; Patsinghasanee et al., 2017). Figure 14 shows the spatial-averaged bank width obtained from the numerical results with and without slump block, together with those obtained from the experimental results. The numerical results with the slump block present good agreement with the experimental results. The results clearly show that cantilever failures are suppressed by effects resulting from the presence of the slump block. In contrast, the numerical results without the slump block cannot replicate the complex mechanism of cantilever failure. Therefore, the slump block on a cantilever failure model significantly affects our ability to replicate the coupling of fluvial erosion and cantilever failure.

For the present numerical model, Fig. 15a illustrates the model capabilities at the cantilever failure stages in which a slump block is present, using cross-sectional views at 300 s. This figure shows the occurrence of fluvial erosion at the slump block and the bank in the submerged zone. This process occurs through fluvial erosion at the slump block and through the undermining of the bank. In the present stage, the slump block was eroded on three sides (left, right, and top). During slump block decomposition, the fluvial erosion rate of the bank is less than the initial rate because of the armouring effect (Dulal & Shimizu, 2010; Motta et al., 2014; Parker et al., 2011). In contrast, in Fig. 15b, the numerical model without the slump block effect cannot replicate the complicated mechanisms of cantilever failure. Additionally, in the numerical results, the fluvial erosion and cantilever failure rates were overestimated.

Finally, beam failure is the dominant failure mechanism in the numerical results. In addition, the widths of the overhanging block in the numerical results are within the range of 5.7 to

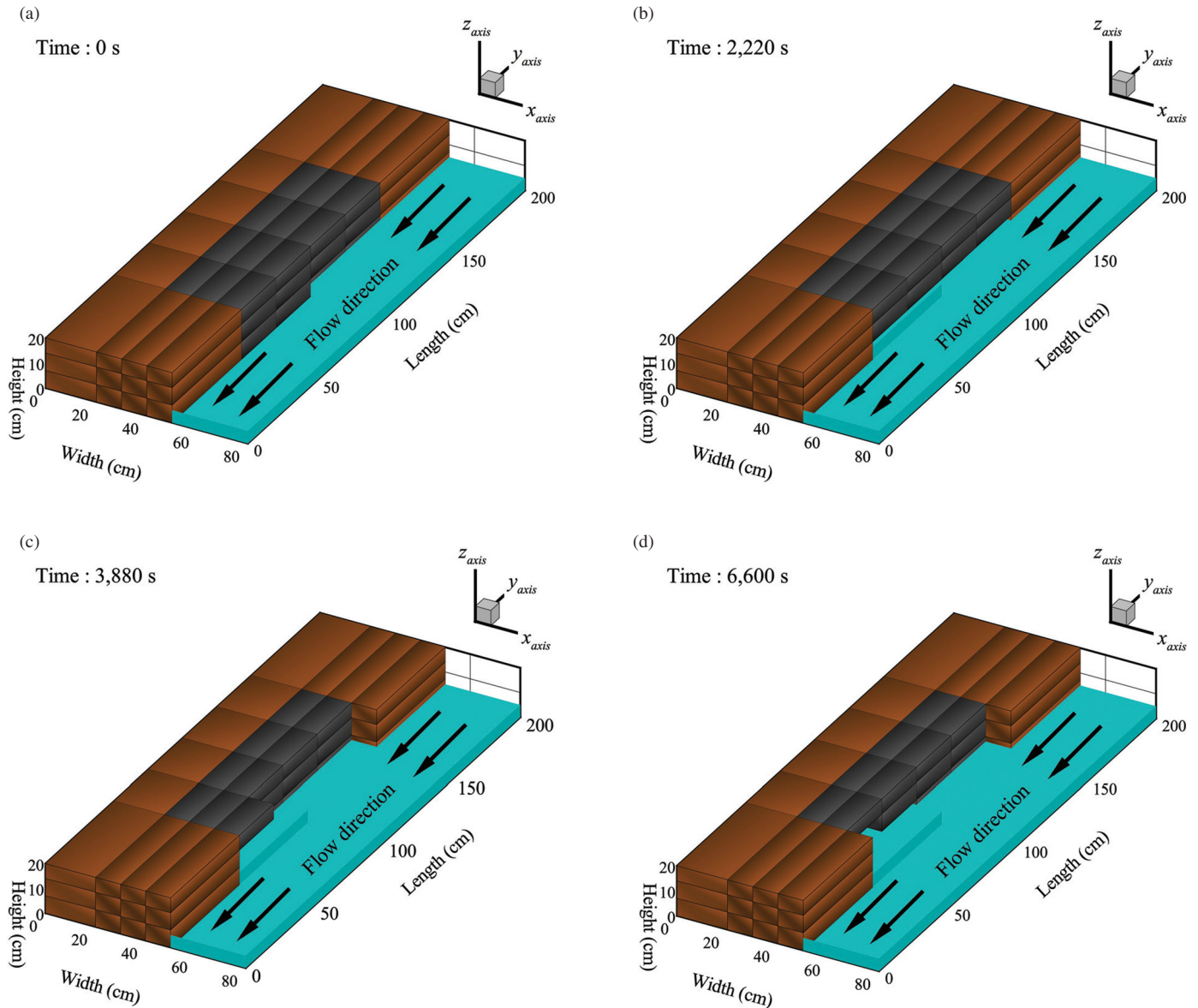


Figure 11 Verified failure processes along the bank in the case of 20% p_{sc} using the acceleration sensors: (a) initial stage; (b) fluvial erosion stage; (c) beam failure stage; (d) equilibrium stage

6.0 cm, while those in the experimental results are within the range of 5.7 to 14.3 cm. Therefore, we can conclude that the simulated results are consistent with the experimental results in terms of the overhanging block dimensions and the dominant failure mechanism.

4 Discussion

4.1 Flow condition

As for the flow conditions, the Reynolds numbers (R) are within the range of 15,000 to 40,000 and the Froude numbers (F) are less than one (Table 1), which indicate that the present flow is in turbulent and sub-critical conditions. In general, the Froude numbers adopted in the present study represent a large number of alluvial rivers where the flow condition is sub-critical

conditions (Dulal & Shimizu, 2010; Patsinghasanee et al., 2017; Peakall, Ashworth, & Best, 2007).

4.2 Fluvial erosion rate

In the experiment, the rate of fluvial erosion is greater in the case of 30% p_{sc} than that in case of 20% p_{sc} under similar hydraulic conditions, as shown by the accumulated bank failure width in Fig. 16. The fluvial erosion rates at the equilibrium stages in the case of 30% and 20% p_{sc} are 0.0025 cm s^{-1} and 0.0014 cm s^{-1} , respectively. The main reason for this is that the banks with higher silt-clay content (lower bank cohesion) are more susceptible to failure than those with a lower content (higher bank cohesion). Additionally, the silt-clay content could be expected to have some bearing on the susceptibility of the banks to sub-aerial erosion, as the cohesive materials with

Table 2 Summary of failure times, failure block dimensions, failure block volumes, and decomposition times (30% of p_{sc})

Cantilever failure	Elapsed time (s)	Failure block dimensions				Decomposition time (s)
		Length (cm)	Width (cm)	Height (cm)	Volume (cm ³)	
I	300	127.3	12.9	14.9	24,468.33	163
*II	461	80.0	14.3	14.7	16,816.80	55
III	1560	64.3	9.9	15.0	9548.55	1763
IV	3300	51.8	8.6	14.5	6459.00	2960
V	5760	54.7	14.2	14.6	11,340.00	2899
VI	6255	39.2	12.8	14.8	7426.05	3981
VII	7200	32.4	10.4	14.8	4987.00	2277
VIII	9420	17.5	11.5	14.7	2958.37	3917

Table 3 Summary of failure times, failure block dimensions, failure block volumes, and decomposition times (20% of p_{sc})

Cantilever failure	Elapsed time (s)	Failure block dimensions				Decomposition time (s)
		Length (cm)	Width (cm)	Height (cm)	Volume (cm ³)	
I	1890	47.7	6.8	14.5	4703.22	289
II	3120	18.4	7.1	14.6	1907.34	61
III	3510	47.3	5.7	14.4	3882.38	175
IV	3880	73.8	10.9	14.7	11,824.97	290
V	4770	45.9	12.3	13.5	7621.69	269
VI	5100	42.1	13.5	13.2	7502.22	904
VII	5505	42.4	8.2	13.5	4693.68	118
VIII	6600	14.8	5.7	14.7	1240.09	107

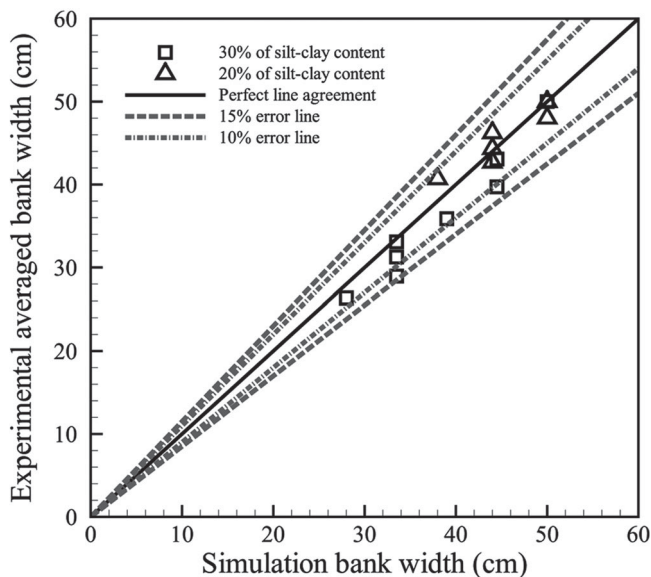


Figure 12 Comparison of numerical and experimental results in the streamwise-averaged bank width

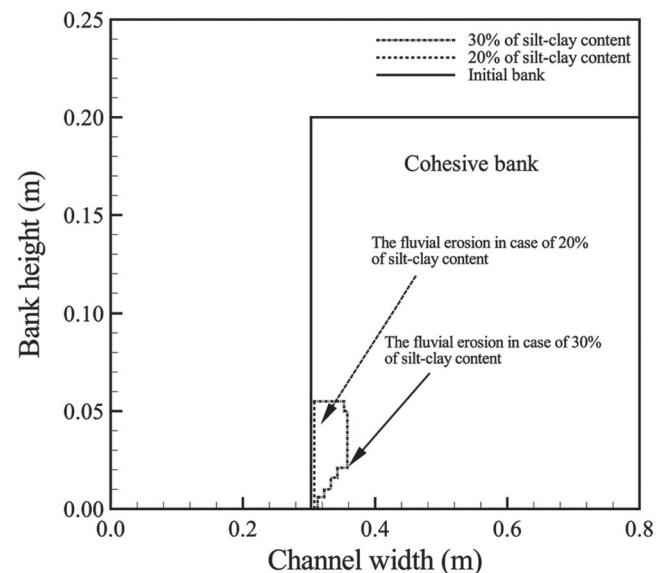


Figure 13 Bank geometry of the numerical results at 270 s

higher silt–clay content (lower bank cohesion) have a greater plasticity (and hence the capacity to swell and shrink) (Couper, 2003; Dulal & Shimizu, 2010; Patsinghasanee et al., 2017). In the numerical analyses, the fluvial erosion rate is greater in the case of 30% p_{sc} than in the case of 20% p_{sc} under similar hydraulic conditions, as shown in Fig. 13. Moreover, the numerical results are consistent with the experimental results in terms

of the qualitative changes of fluvial erosion rate depending on the silt–clay content.

4.3 Backward erosion

The experiments were able to randomly replicate cantilever failures – as illustrated in Fig. 10 – and the experimental results

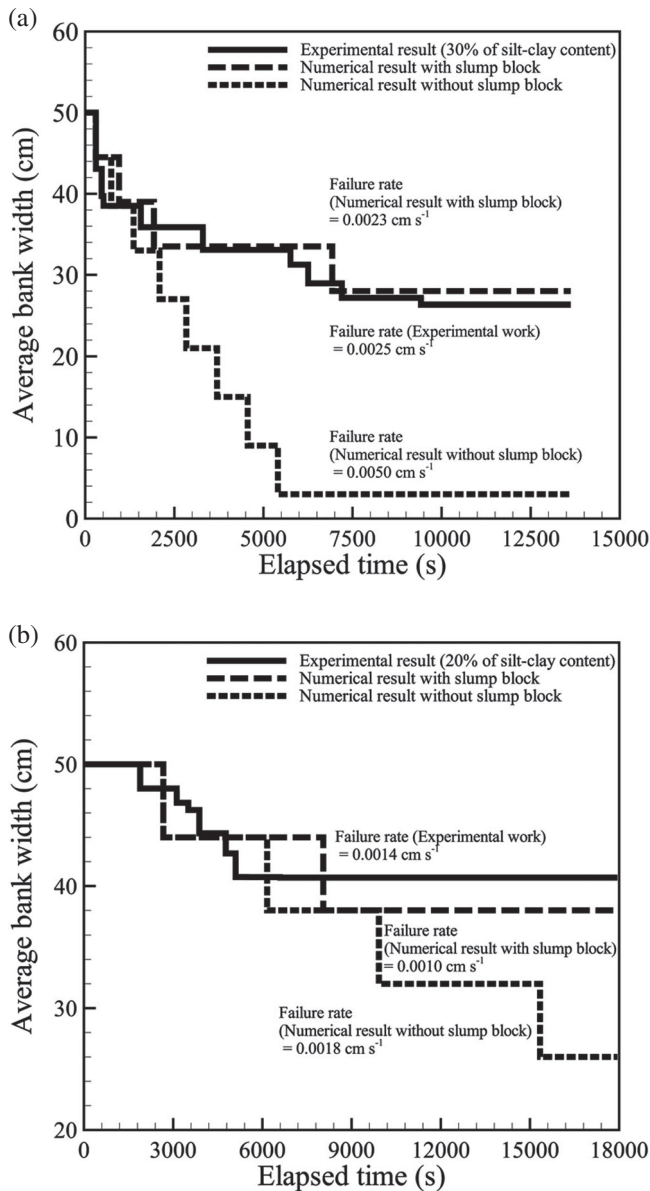


Figure 14 Comparison of spatial-averaged bank width with and without the slump block, along with the experimental results; (a) case of 30% p_{sc} ; (b) case of 20% p_{sc}

show a backward erosion process in which the erosion progresses toward the upstream region of the bank in the case of 20% p_{sc} (high bank cohesion), which is similar to previous observations (Hooke, 1980; Patsinghasanee et al., 2017). In the present study, bank particles detached from the downstream region through the effect of a second wooden panel generating a reverse flow. This process is summarized as follows. The flow from the upstream region collided with the second wood panel downstream of the banks, and the flow direction reversed and the flow velocity was high so the banks eroded upstreamward from the downstream region. For this reason, this study considered only the centre region and used streamwise-averaged data of banks to reduce the effect of backward erosion.

4.4 Slump block

In the present study, slump block formation and decomposition were observed and recorded in detail on the laboratory scale. The behaviour of slump blocks seems highly complex, and the slump block behaviour is highly significant to the cantilever failure process at the experimental scale. In the present study, the effect of the slump block behaviour on cantilever failure was observed, as was its role in such failure, and it was described for the first time at the experimental scale. Moreover, the effect of slump blocks formation on the bed channel in front of the bank, where they formed a sediment buffer that reinforced the bank and reduced fluvial erosion, was observed. The average slump block widths, lengths, and heights for two different cohesions of the bank are presented in Fig. 17, which shows a clear tendency for the dimensions to be lower in the case of higher bank cohesion. Therefore, the geometry of the slump block, as well as its failure and decomposition, mainly depend on the cohesion of the bank. A reduction in the cohesion of the bank not only increases the fluvial erosion rate, but also affects the dimensions of the slump blocks and their decomposition rates (Figs 16 and 17). The observed phenomena in the present work are consistent with previous studies in terms of the changes in bank cohesion (Couper, 2003; Dulal & Shimizu, 2010; Julian & Torres, 2006; Patsinghasanee et al., 2017). Figure 18 shows the relationship between the volumes of the slump blocks and their decomposition times in the present study. This relation seems to be largely random, without any identifiable rules governing this phenomenon, because the decomposition process is dependent on processes such as weathering, fluvial erosion, general disintegration, and removal (Wood, Simon, Downs, & Thorne, 2001). This random relationship between the volumes of the slump blocks and their decomposition times has been reported in previous observations of meandering experiments with clay-mixed sediment (Dulal & Shimizu, 2010).

4.5 Armouring effect

Additionally, the armouring effect of the slump block was revealed in the experiments. The detached and re-suspended sand particles from the slump block sank again and deposited in front of the bank-toe, which caused the formation of a surface layer of failure materials during the experimental run (Fig. 19). The strength of the bank-toe progressively increased during the experimental run owing to the continuous accumulation of cohesive material that was generated as suspended load from the decomposed slump block. Furthermore, the temporal increase in armouring effect was observed at the bank-toe due to greater segregation of the cohesive material. After the quasi-equilibrium stage, the armouring effect became stronger at locations of greater silt-clay content. Therefore, this mechanism caused the strength of the bank-toe to increase.

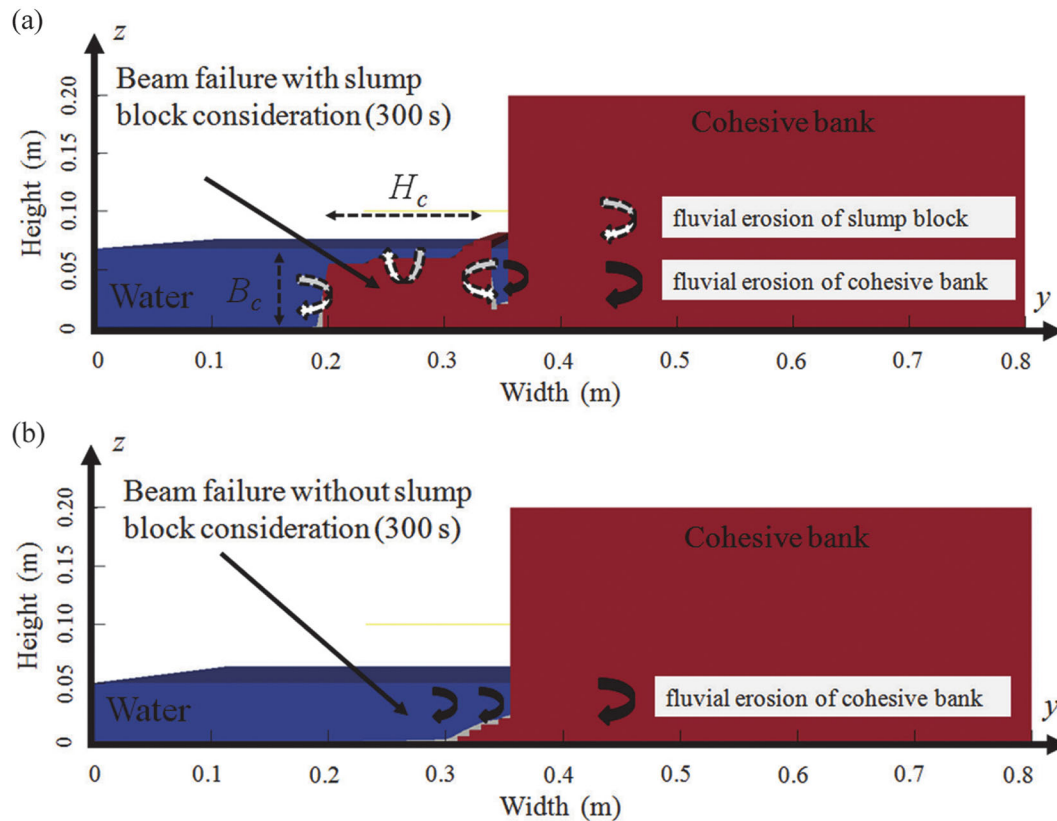


Figure 15 Cross-sectional views of computational results in the case of 30% p_{sc} (a) with the slump block; and (b) without the slump block (brown: undisturbed cohesive material; grey: non-cohesive bedload; blue: water)

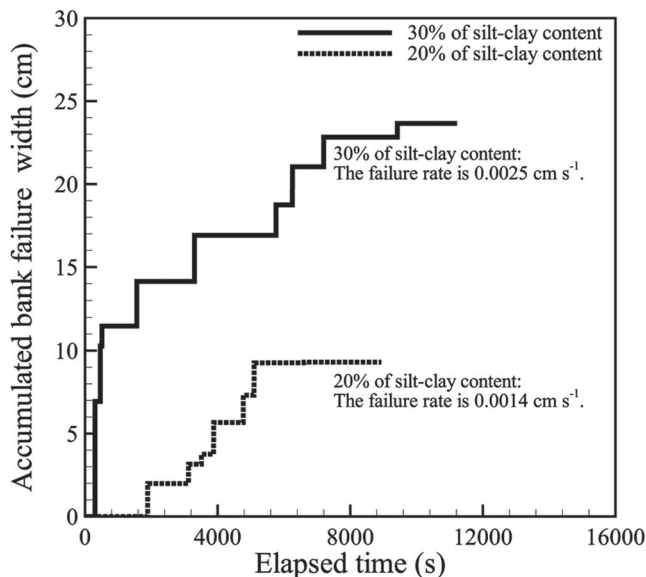


Figure 16 Results of fluvial erosion rate in the experimental studies with different silt-clay contents

5 Conclusions

This study elucidated the complex mechanisms of cantilever failure, considering the slump block by means of experiments and modelling. In the laboratory experiments, two types of cohesive materials with different percentages of silt-clay content

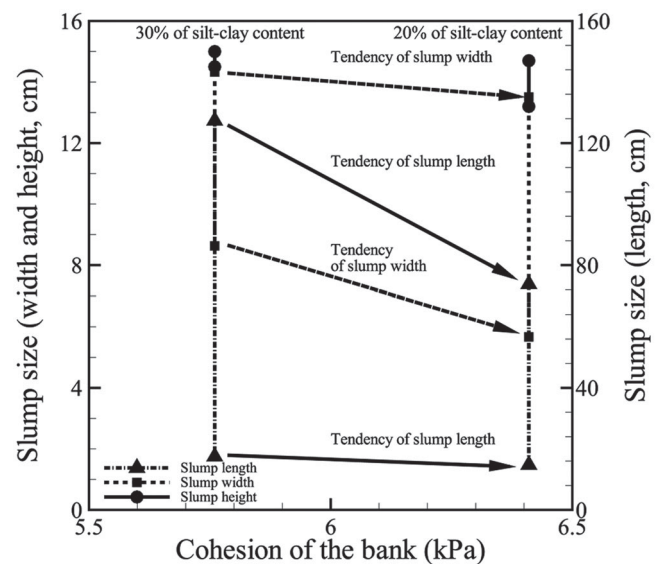


Figure 17 Variation of slump block dimensions with different bank cohesion (dot-dash and dashed lines show linear decreases in slump block length and width, respectively)

were investigated under similar hydraulic conditions, using acceleration sensors to clarify the cantilever processes. The novel modelling of cantilever failure that was realized using a triple-grid approach within the framework of fluvial erosion, the cantilever's subsequent failure, the slump block behaviour, and sediment transport was validated by the experimental

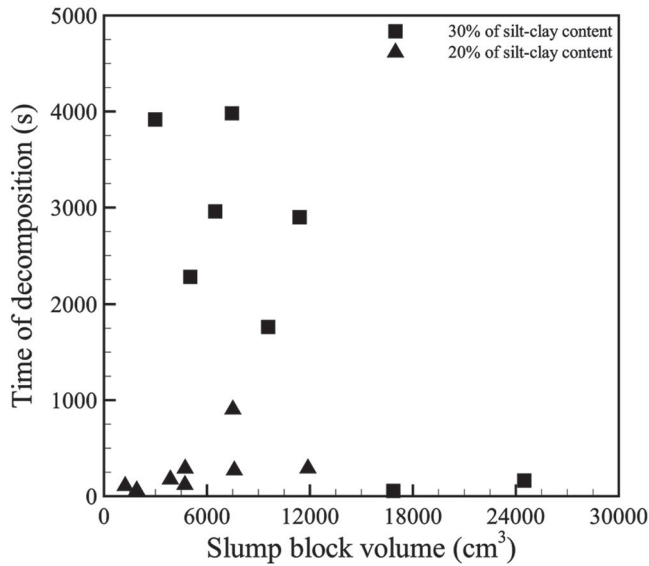


Figure 18 Slump decomposition of the various volumes with different bank cohesion

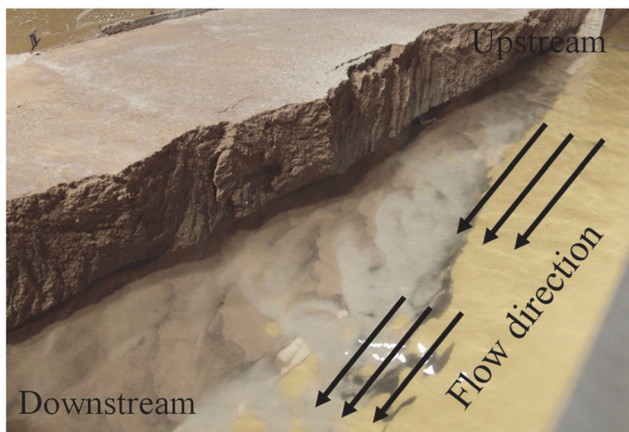


Figure 19 Armouring effect observed in the experiment in the case of 20% p_{sc}

results. The main conclusions from the results are as follows.

For the complex mechanism of cantilever failure, the experimental results showed that fluvial erosion at the submerged zone generated an overhanging block in the upper part of the banks. Tension cracks then developed on the upper surface of the bank, and cantilever failure occurred along the tension crack lines. The dominant failure mechanism was observed to be beam failure, which was verified using the acceleration sensors installed in the banks. The mechanisms of cantilever failure are consistent with the findings of previous research (e.g. Patsinghasanee et al., 2017; Pizzuto, 1984a, 1984b; Samadi et al., 2013; Thorne & Tovey, 1981; Van Eerd, 1985). In addition, the experimental results indicate that banks with lower cohesion are more susceptible to failure than those with higher cohesion. These experimental observations are similar to those of previous works in terms of changes in bank cohesion (Couper, 2003; Dulal & Shimizu, 2010; Julian & Torres, 2006; Patsinghasanee et al.,

2017). The slump block failure and decomposition phenomena were discussed along with the cantilever failure experiments on channel flows. Slump blocks were observed in front of the bank-toe, where they formed a sediment buffer that reinforced the banks and reduced fluvial erosion. The formation and deformation of the slump block had a significant effect on bank cohesion. Therefore, a reduction in bank cohesion led to larger slump block dimensions, as well as slower decomposition.

The modelling satisfactorily reproduced fluvial erosion, cantilever failure, and bank protection by the slump blocks. Moreover, the numerical results presented good agreement with the experimental results in terms of the spatial-averaged bank width. In contrast, the numerical results without the slump block presented fluvial erosion and cantilever failure rates that were greater than those of the experimental results. Therefore, the slump block's bank protection effects are clearly demonstrated in this study. Additionally, the numerical results in terms of the cantilever failure mechanism are similar to the experimental results, demonstrating that beam failure is the dominant failure mechanism. In addition, the widths of the overhanging blocks in the numerical results are consistent with those in the experimental results. We can conclude that the present numerical model is a powerful tool for the analysis and prediction of the complex mechanism of cantilever failure with slump blocks. The present numerical model has limitations in simulating the complex flow characteristics near the banks, the non-uniform cantilever failure, the longitudinal gradient of sediment transport, and the pore water pressure. The proposed model might be important for guiding future research and for comparison with data from natural rivers in terms of cantilever failure with a slump block.

Acknowledgements

The authors would like to express their sincere gratitude to the Japanese Government Scholarships for the financial support of scholarships. The acceleration sensors were supplied by the Obihiro Regional Office of Hokkaido Development Bureau.

Notation

A_b	= cross-sectional area of overhanging block (m^2)
A_j	= cross-sectional area in each calculation cell (m^2)
a	= an exponent value (—)
B_0	= initial bank width (m)
B_c	= overhanging block width (m)
C	= bank cohesion (Pa)
D	= water depth (m)
D_0	= initial water depth (m)
D_{50}	= mean diameter (m)
f_d	= driving force acting on the overhanging block (N)
f_r	= resisting force acting on the cohesive bank (N)

F	=	Froude number (—)
f_{sb}	=	factor of safety of beam failure (—)
f_{ss}	=	factor of safety of shear failure (—)
g	=	gravitational acceleration (m s^{-2})
H_0	=	initial bank height (m)
H_c	=	overhanging block height (m)
H_d	=	effective length (m)
i_w	=	plasticity index (—)
i	=	bed slope (—)
j	=	number of the lateral cell (—)
k_d	=	erodibility coefficient ($\text{m}^3 \text{N}^{-1} \text{s}^{-1}$)
k_s	=	roughness height (m)
k_{sb}	=	coefficient of the fluvial erosion rate (—)
L_c	=	overhanging length under compression (m)
L_t	=	overhanging length under tension (m)
n	=	Manning roughness parameter along the channel ($\text{m}^{-1/3} \text{s}$)
P_j	=	perimeter length in each calculation cell (m)
Q	=	discharge (l s^{-1})
q_{bn}	=	sediment transport rate in the lateral direction ($\text{m}^2 \text{s}^{-1}$)
q_{bs}	=	sediment transport rate in the streamwise direction ($\text{m}^2 \text{s}^{-1}$)
R	=	Reynolds number (—)
R_j	=	hydraulic radius in each calculation cell (m)
p_{sc}	=	percentage of silt–clay content (—)
s_g	=	specific weight of sediment (—)
t	=	time (s)
u_*	=	shear velocity (m s^{-1})
u_j	=	velocity in each calculation cell (m s^{-1})
V_{erd}	=	simulated fluvial erosion volume in each timestep (m^3)
V_{org}	=	initial grid cell volume of a bank (m^3)
w	=	overhanging block weight (N)
p_{wc}	=	water content (—)
x_{axis}	=	acceleration value in x direction (m s^{-2})
y	=	coordinate component in the lateral direction (m)
y_{axis}	=	acceleration value in the y direction (m s^{-2})
z_{axis}	=	acceleration value in the z direction (m s^{-2})
Z_b	=	bed elevation in each calculation cell (m)
Z_c	=	tension crack depth (m)
ε	=	fluvial erosion rate (m s^{-1})
ε_{sb}	=	fluvial erosion rate of the slump block (m s^{-1})
γ_s	=	unit weight of soil (N m^{-3})
λ	=	porosity of the material (—)
μ_k	=	kinetic friction coefficient (—)
μ_s	=	static friction coefficient (—)
ρ	=	density of water (kg m^{-3})
θ	=	angle of repose ($^\circ$)

σ_c	=	uniform compressive strength (Pa)
σ_t	=	uniform tensile strength (Pa)
ξ	=	ratio of tensile strength to cohesion (—)
τ_*	=	non-dimensional bed shear stress (—)
τ_c^*	=	non-dimensional critical bed shear stress (—)
τ_{b0}	=	actual shear stress applied by the flow (Pa)
τ_{bc}	=	critical shear stress (Pa)
ϕ	=	internal friction angle ($^\circ$)
Ω_t	=	undisturbed cell occupancy rate (—)

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